

## Week 3

### Topic: Three Dimensional Shapes

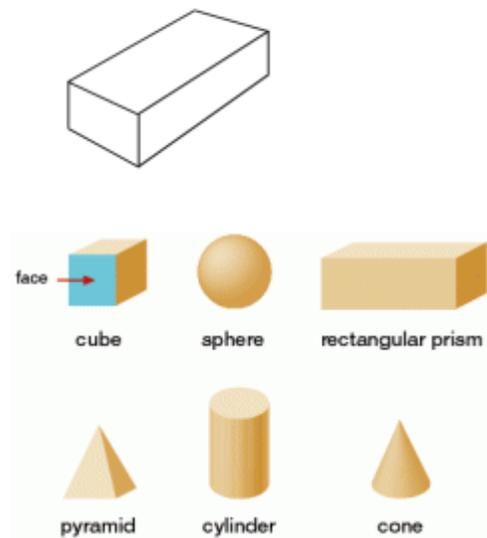
#### Three-dimensional shapes

Nearly everything that we can see and touch takes up space. These things are either **gases, liquids or solids**. You will study some of the properties of liquids and gases in science.

Most solids, or **three-dimensional shapes**, such as stones and trees, have rough and **irregular** shape. This usually occurs in nature. However, some three-dimensional shapes, such as tin, can and houses, have **regular** shapes. These are usually made by people. We often call them **geometrical solids**.

Look at the photograph below, it shows groundnut pyramids

The diagram below shows that each pyramid is made up from a square-based pyramid and a cuboid.



#### Cuboids and cubes

Look at a matchbox. The name of its shape is a **cuboid**

A face may be flat (**plane**) or curved. A cuboid has 6 plane faces. Each face is in the shape of a rectangle.



A rectangle

An edge is a line where two faces meet. It may be straight or curved. A cuboid has 12 straight edges.

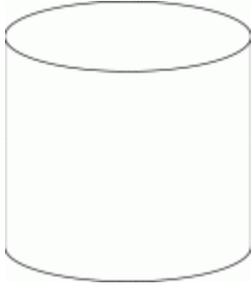
A **vertex** is a point or corner where three or more edges meet. The plural of vertex is **vertices**. A cuboid has 8 vertices.

### **Drawing cuboids**

There are many ways of drawing cuboids. In each case it is impossible to show the solid as it really is. A **skeleton view** is very useful since it shows all the edges. The drawings below show how to draw a skeleton view very quickly. Notice that some edges are hidden from view. We usually show these as broken lines.

### **Cubes**

A cube is a cuboid in which all six faces are squares

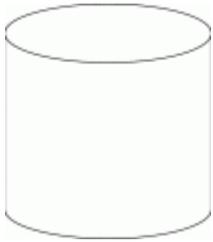


Exercise

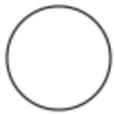
Write down five everyday objects which are cuboids

### **Cylinders and prisms**

#### **Cylinder**



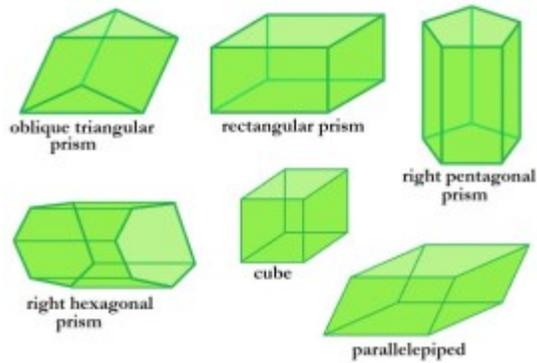
The cylinder has two plane faces and one curved face. It has no vertices and two curved edges> the two plane faces are both circles.



A skeleton view of a cylinder is drawn in much the same way as that of a cuboid

#### **Prism**

The base and top faces of a prism are always the same shape. The names of prisms come from the shape of their base and top faces. The side faces of right prisms are always rectangular.



The cuboid is a rectangular prism and the cylinder can be thought of as a special prism.

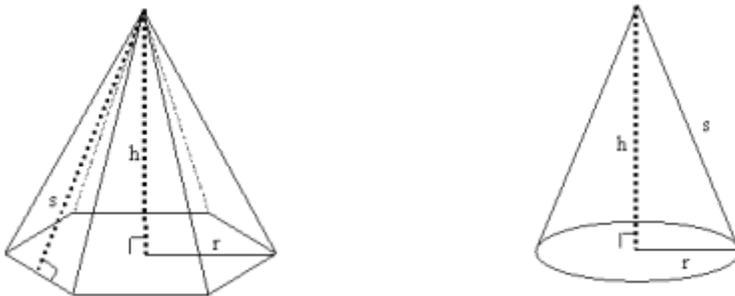
### **Cones and Pyramids**

**Pyramid:** A 3-dimensional solid in which the base is a polygon and the sides are triangles which meet in one point called the vertex. We shall examine regular pyramids in which the base is a regular polygon and the sides are congruent triangles.

**A Right Circular Cone:** A 3-dimensional solid in which the base is a circle. The side of a cone is formed by straight lines which connect the circular base to a vertex. The height is the perpendicular distance from the vertex to the base and meets the base in the center of the circle.

### **The Lesson:**

The diagrams below show a pyramid and a cone. Both have a height of  $h$  and radius of  $r$ . In the pyramid at left,  $r$  is the radius of the regular hexagon that is the base of the pyramid. In the cone at right,  $r$  is the radius of the circular base. The slant height is  $s$  in both diagrams.



### **Sphere**

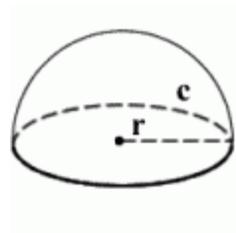
Nearly every ball is sphere shaped



A tennis ball

Outline of a ball; sphere

Half of a sphere is called a hemisphere



### Exercise

Write down five everyday objects each of which is either sphere-shaped or contains part of a sphere in its shape.

Which of the following solids will roll smoothly on a plane surface?

a. cube b. cone c. sphere d. cylinder e. cuboid

### SOLIDS 2: VOLUME

The volume of a solid is a measure of the space it takes up. The cube is used as the shape for the basic unit of volume. A cube of edge 1 metre has a volume of **1 cube metre** or **1 m<sup>3</sup>**. A cube of edge 1 centimetre has a volume of **1 cubic centimeter** or **1 cm<sup>3</sup>**.

It is different to measure volume directly. One way is to build a copy of the solid using basic units. For example, to measure the volume of the 6 cm by 3 cm by 4 cm cuboid in the figure below, a copy can be built from 1 cm<sup>3</sup> cubes.

Units of volume

The **cubic metre, m<sup>3</sup>**, is the basic unit of volume.

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m}^3 = (100 \times 100 \times 100) \text{ cm}^3 = 1\,000\,000 \text{ cm}^3$$

Similarly

$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ cm}^3 = (10 \times 10 \times 10) \text{ mm}^3 = 1\,000 \text{ mm}^3$$

When calculating problems about volume, make sure that all dimension are in the same units.

Volume of Cuboids and Cubes

Cuboid

Notice that

a. the 6 cm by 3 cm by 4 cm cuboid in the above figure has a volume of  $72 \text{ cm}^3$

b.  $6 \times 3 \times 4 = 72$ .

We can the volume of any cuboid by finding the product of its length, breadth and height:

**volume of cuboid = length X breadth X height**

**= area of base X height**

**= area of end face X length**

**= area of side face X breadth**

Cube

Similarly, for a cube of side  $s$ ,

Volume of cube = (side) X (side) X (side)

$$= s \times s \times s$$

$$= s^3$$

Example 1

Calculate the volume of a rectangular box which measures 30 cm X 15 cm X 10 cm.

$$\text{Volume of box} = (30 \times 15 \times 10) \text{ cm}^3$$

$$= 4\,500 \text{ cm}^3$$

### Example 2

A rectangular room 4 m long by 3 m wide contains  $30 \text{ m}^3$  of air.

Calculate the height of the room.

$$\text{Volume of room} = 30 \text{ m}^3$$

$$\text{area of floor (base)} = 4 \text{ m} \times 3 \text{ m} = 12 \text{ m}^2$$

$$\text{height of room} = 30/12 \text{ m} = 2\frac{1}{2} \text{ m}$$

### Capacity of containers

The **capacity of a container** is the measure of the space inside it. The basic unit of capacity is the **litre**. 1 litre of water will just fill a 10 cm by 10 cm by 10 cm cubic container.

Therefore in practice,

$$1 \text{ liter} = (10 \times 10 \times 10) \text{ cm}^3 = 1\,000 \text{ cm}^3$$

The table below shows the relation between units of capacity and units of volume.

	capacity	Volume
Kilolitre	1 kl = 1 000l	= 1 000 000 $\text{cm}^3$ = 1 $\text{m}^3$
Litre	1 l	= 1 000 $\text{cm}^3$
Milliliter	1 ml = 0.001 l	= 1 $\text{cm}^3$

### Example

How many litres of water does a 5 m X 4 m X 3 m tank hold?

$$\text{volume of tank} = (5 \times 4 \times 3) \text{ m}^3 = 60 \text{ m}^3$$

$$\text{but, } 1 \text{ m}^3 = 1\,000 \text{ litres}$$

$$\text{capacity of tank} = 60 \times 1\,000 \text{ litres}$$

$$= 60\,000 \text{ litres}$$

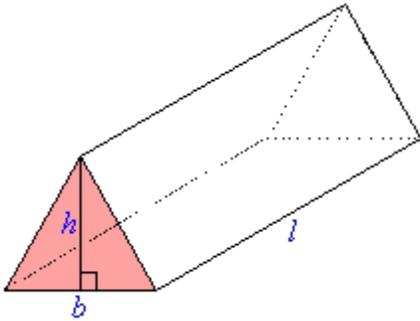
### ASSESSMENT

1. A rectangular tin measure 10 cm by 10 cm by 20 cm. What is its capacity in litres?
2. Calculate the capacity in litres of a tin 20 cm by 2 cm by 10 cm.

### Volume of right-angled triangular prism

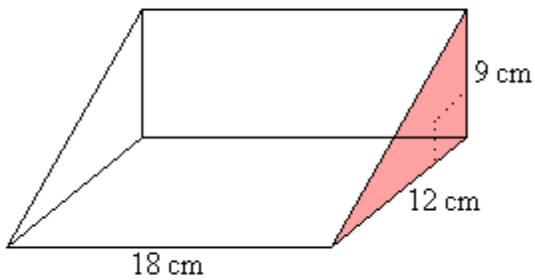
A triangular prism whose length is  $l$  units, and whose triangular cross-section has base  $b$  units and height  $h$  units, has a volume of  $V$  cubic units given by

$$v = Al = \frac{1}{2} bhl$$



3.

Find the volume of the triangular prism shown in the diagram.



$$V = Al$$

$$= \frac{1}{2} bhl$$

$$= \frac{1}{2} \times 12 \times 9 \times 18$$

$$= 972$$