

# Topic: CONSTRUCTION: PARALLEL AND PERPENDICULAR

## LINES

In geometry, to **construct** a figure means to draw it accurately. Accurate construction depends on using measuring and drawing instruments properly.

There is a useful property to show that 2 given lines are parallel. This property states that if 2 given lines are both perpendicular to a third line, then the 2 lines are parallel. The figure below illustrates this property.

### **Example:**

Construct a line parallel to  $AB$  and 2 cm above it.

### **Solution:**

**Step 1:** Mark a point  $C$  anywhere on the line  $AB$ . Construct a line perpendicular to  $AB$  and passing through  $C$ .

**Step 2 :** Construct a line segment on the perpendicular line 2 cm above  $C$ . Label the point as  $D$ . Then, construct a line perpendicular to  $CD$  and passing through  $D$ . This line is parallel to  $AB$ .

### **Constructing Perpendiculars**

Construct a perpendicular from a point on a line

Given: Point  $P$  is on a given line

Task: Construct a line through  $P$  perpendicular to the given line.

Directions

1. Place your compass point on  $P$  and sweep an arc of any size that crosses the line twice (below the line). You will be creating (at least) a semicircle. (Actually, you may draw this arc above OR below the line.)
2. STRETCH THE COMPASS LARGER!!
3. Place the compass point where the arc crossed the line on one side and make a small arc below the line. (The small arc could be above the line if you prefer.)

4. Without changing the span on the compass, place the compass point where the arc crossed the line on the OTHER side and make another arc. Your two small arcs should be crossing.
5. With your straightedge, connect the intersection of the two small arcs to point  $P$ .

This new line is perpendicular to the given line.

Explanation of construction: Remember the construction for bisect an angle? In this construction, you have bisected the straight angle  $P$ . Since a straight angle contains 180 degrees, you have just created two angles of 90 degrees each. Since two right angles have been formed, a perpendicular exists.

Construct a perpendicular to a line from a point on a line

**Given:** Point  $P$  is off a given line

**Task:** Construct a line through  $P$  perpendicular to the given line.

Directions

1. Place your compass point on  $P$  and sweep an arc of any size that crosses the line twice.
2. Place the compass point where the arc crossed the line on one side and make an arc ON THE OPPOSITE SIDE OF THE LINE.
3. Without changing the span on the compass, place the compass point where the arc crossed the line on the OTHER side and make another arc. Your two new arcs should be crossing on the opposite side of the line.
4. With your straightedge, connect the intersection of the two new arcs to point  $P$ .

This new line is perpendicular to the given line.

Explanation of construction: To understand the explanation, some additional labeling will be needed. Label the point where the arc crosses the line as points  $C$  and  $D$ . Label the intersection of the

new arcs on the opposite side as point  $E$ . Draw segments  $PC$ ,  $PD$ ,  $EC$ , and  $ED$ . By the construction,  $PC = PD$  and  $EC = ED$ . Now, remember a locus theorem: The locus of points equidistant from two points ( $C$  and  $D$ ), is the perpendicular bisector of the line segment determined by the two

points. Hence,  $PE$  is the perpendicular bisector of  $CD$ .

The fact that we created a bisector, as well as a perpendicular, is actually MORE than we needed – we only needed to create a perpendicular.