WEEK 1 & 2

Topic: REFRACTION OF LIGHT

INTRODUCTION

When a ray travels from one transparent medium to another of different density, its direction is abruptly changed at the surface separating the two media. This is known as the refraction of the light ray. Thus a light ray appears to bend as it crosses the boundary of two different media. Refraction is due to the difference in the speed of light in the different media.

Refraction is the bending of a light ray as it crosses the boundary between two media of different densities, thus causes a change in direction.

The phenomenon of refraction is responsible for the following common observations: (i) The bottom of a clear river or pond appears shallower than it really is. (ii) A rod or spoon appears bent or broken when it is partially immersed in water or any liquid. (iii) Letters in prints seem to be nearer when we place a thick block of glass over them.

Laws of Refraction

Two laws are associated with refraction. The first law of refraction states that, the incidence, and the refracted ray all lie in the same plane.

The second law states that, the ratio of sine of the angle of incidence to the sine of the angle of refraction is constant for all rays passing from one medium to another.

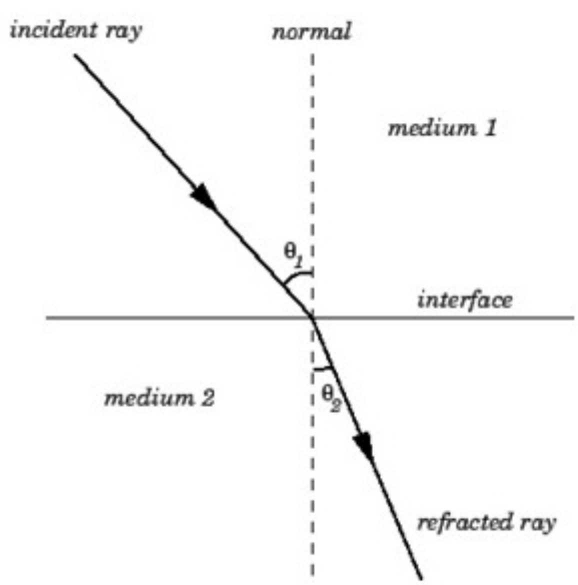
The two lays of refraction were postulated by a physicsist called Snell. Snell's first law of refraction is given as, Sin *I*/Sin r = n, a constant, for a given pair of media.

The law of refraction, which is generally known as *Snell's law*, governs the behaviour of light-rays as they propagate across a sharp interface between two transparent dielectric media.

Consider a light-ray incident on a plane interface between two transparent dielectric media, labelled 1 and 2, as shown in the Fig below. The law of refraction states that the incident ray, the refracted ray, and the normal to the interface, all lie in the *same plane*. Furthermore,

$n_1 \sin \theta_1 = n_2 \sin \theta_2$,

where θ_1 is the angle subtended between the incident ray and the normal to the interface, and θ_2 is the angle subtended between the refracted ray and the normal to the interface. The quantities n_1 and n_2 are termed the *refractive indices* of media 1 and 2, respectively. Thus, the law of refraction predicts that a light-ray always deviates more towards the normal in the optically denser medium: *i.e.*, the medium with the higher refractive index. Note that $n_1 > n_2$ in the figure. The law of refraction also holds for non-planar interfaces, provided that the normal to the interface at any given point is understood to be the normal to the local tangent plane of the interface at that point.



By definition, the refractive index of a dielectric medium of dielectric constant is given by

$n = \sqrt{K}$.

Table below shows the refractive indices of some common materials (for yellow light of wavelength λ = 589nm).

Refractive Indices of some common materials at $\lambda = 589$nm.

Material	n
Air (STP)	1.00029
Water	1.33
Ice	1.31
Glass:	
Light Flint	1.58
Heavy Flint	1.68
Heaviest Flint	1.89
Diamond	2.42

The law of refraction follows directly from the fact that the speed v with which light propagates through a dielectric medium is *inversely proportional* to the refractive index of the medium, $v = c/n$, where c is the speed of light in a vacuum. Consider two parallel light-rays, a and b, incident at an angle θ_1 with respect to the normal to the interface between two dielectric media, 1 and 2. Let the refractive indices of the two media be n_1 and n_2 respectively, with $n_2 > n_1$. It is clear from the figure below that ray b must move from point B to point Q, in medium 1, in the same time interval, Δt, in which ray *a*moves between points A and P, in medium 2. Now, the speed of light in medium 1 is $v_1 = c/n_1$, whereas the speed of light in medium 2 is $v_2 = c/n_2$. It follows that the length BQ is given by $v_1 \Delta t$, whereas the length AP is given by $v_2 \Delta t$. By trigonometry,

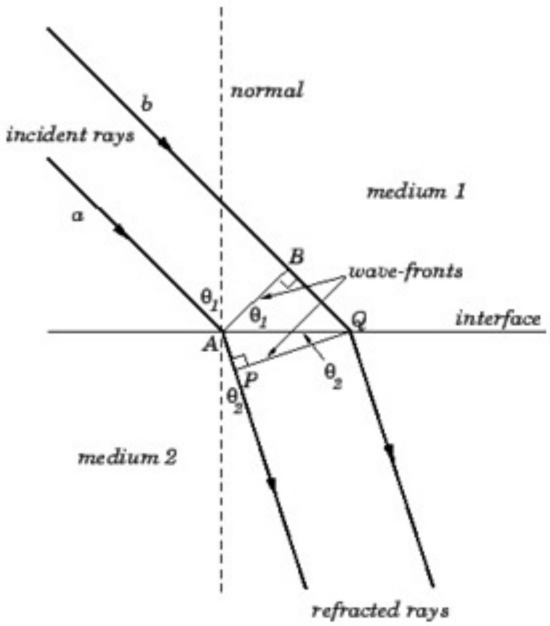
$\sin \theta_1$ BQ/AQ = $v_1 \Delta t$/AQ,

and

$\sin \theta_2$ AP/AQ = $v_2 \Delta t$/AQ

Hence $\sin \theta_1 / \sin \theta_2 = v_1/v_2 = n_2/n_1$

which can be rearranged to give Snell's law. Note that the lines AB and PQ represent wave-fronts in media 1 and 2, respectively, and, therefore, cross rays and at right-angles.



Derivation of Snell's law

When light passes from one dielectric medium to another its velocity changes, but its frequency f remains *unchanged*. Since, $v = f\lambda$ for all waves, where λ is the wavelength, it follows that the wavelength of light must also change as it crosses an interface between two different media. Suppose that light propagates from medium 1 to medium 2. Let n_1 and n_2 be the refractive indices of the two media, respectively. The ratio of the wave-lengths in the two media is given by

$\lambda_2/\lambda_1 = v_2/f \,/\, v_1/f = v_2/v_1 = n_2/n_1$

Thus, as light moves from air to glass its wavelength *decreases.*

Again, the constant n, is known as the refractive index of the second medium with respect to the first medium. It is a number which gives a measure of refraction or bending of light as it travels from one medium to another. If light is travelling from air to glass, the refractive index of glass is given by

${}_an_g$ = sine of angle of incidence in air/sine of angle of refraction in glass

If light travels from glass to air then the refractive index ${}_gn_a$ = sine of angle of incidence in glass/sine of angle of refraction in air

From the principle of the reversibility of light we have:

${}_an_g = 1 / {}_gn_a$

Since refraction is due to the change in the speed of light as it travels from one medium to another, the refractive index is also given by

${}_an_g$ = speed of light in air (vacuum)/speed of light in glass

Terms Associated with Refraction

(i) The incident ray: This is the direction of rays of the light from the source to the first medium.

(ii) The refracted ray: This is the direction to which the light travels from the point of incidence to the second medium which is always denser than the first medium.

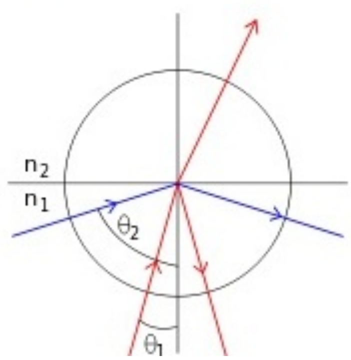
(iii) The angle of incidence: Thus is the angle at which the incident ray mixes with the normal in the first medium.

(iv) The angle of refraction: This is the angle at which the refracted ray mixes with the normal in the second medium.

Total Internal Reflection

Total internal reflection is a phenomenon that happens when a propagating wave strikes a medium boundary at an angle larger than a particular critical angle with respect to the normal to the surface. If the refractive index is lower on the other side of the boundary and the incident angle is greater than the critical angle, the wave cannot pass through and is entirely reflected. The **critical angle** is the angle of incidence above which the total internal reflectance occurs. This is particularly common as an optical phenomenon, where light waves are involved, but it occurs with many types of waves, such as electromagnetic waves in general or sound waves.

When a wave crosses a boundary between materials with different kinds of refractive indices, the wave will be partially refracted at the boundary surface, and partially reflected. However, if the angle of incidence is greater (i.e. the direction of propagation or ray is closer to being parallel to the boundary) than the critical angle – the angle of incidence at which light is refracted such that it travels along the boundary – then the wave will not cross the boundary and instead be totally reflected back internally. This can only occur when the wave in a medium with a higher refractive index (n_1) hits its surface that's in contact with a medium of lower refractive index (n_2). For example, it will occur with light hitting air from glass, but not when hitting glass from air.



The larger the angle to the normal, the smaller is the fraction of light transmitted rather than reflected, until the angle at which total internal reflection occurs. (The color of the rays is to help distinguish the rays, and is not meant to indicate any color dependence).

The critical angle is the angle of incidence *above* which total internal reflection occurs. The angle of incidence is measured with respect to the normal at the refractive boundary (see diagram illustrating Snell's law). Consider a light ray passing from glass into air. The light emanating from the interface is bent towards the glass. When the incident angle is increased sufficiently, the transmitted angle (in air) reaches 90 degrees. It is at this point no light is transmitted into air. The critical angle θ_c is given by Snell's law,

$n_1 \sin \theta_i = n_2 \sin \theta_t$.

Rearranging Snell's Law, we get incidence

$\sin \theta_i = n_2/n_1 \sin \theta_t$

To find the critical angle, we find the value for θ_i when $\theta_t = 90°$ and thus $\sin \theta_t = 1$. The resulting value of θ_i is equal to the critical angle θ_c.

Now, we can solve for θ_i, and we get the equation for the critical angle:

$\theta_c = \theta_i = \arcsin (n_2/n_1)$,

If the incident ray is precisely at the critical angle, the refracted ray is tangent to the boundary at the point of incidence. If for example, visible light were traveling through acrylic glass (with an index of refraction of approximately 1.50) into air (with an index of refraction of 1.00), the calculation would give the critical angle for light from acrylic into air, which is

$\theta_c = \arcsin (1.00/1.50) = 41.8^o$

Light incident on the border with an angle less than 41.8° would be partially transmitted, while light incident on the border at larger angles with respect to normal would be totally internally reflected.

If the fraction n_2/n_1 is greater than 1, then arcsine is not defined—meaning that total internal reflection does not occur even at very shallow or grazing incident angles.

So the critical angle is only defined when n_2/n_1 is less than 1.

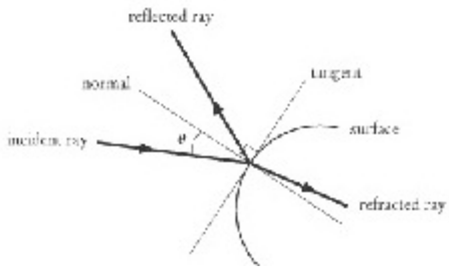
Refraction of light at the interface between two media, including total internal reflection.

A special name is given to the angle of incidence that produces an angle of refraction of 90˚. It is called the critical angle.

Topic: LENSES

INTRODUCTION

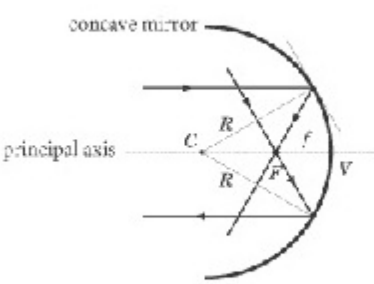
The reflection and refraction we have dealt with so far have focused only on light interacting with flat surfaces. Lenses and curved mirrors are optical instruments designed to focus light in predictable ways. While light striking a curved surface is more complicated than the flat surfaces we have looked at already, the principle is the same. Any given light ray only strikes an infinitesimally small portion of the lens or mirror, and this small portion taken by itself is roughly flat. As a result, we can still think of the normal as the line perpendicular to the tangent plane.



The four basic kinds of optical instruments—the only instruments are concave mirrors, convex mirrors, convex (or converging) lenses, and concave (or diverging) lenses. If you have trouble remembering the difference between concave and convex, remember that, like caves, concave mirrors and lenses curve inward. Convex lenses and mirrors bulge outward.

General Features of Mirrors and Lenses

Much of the vocabulary we deal with is the same for all four kinds of optical instruments. Before we look at the peculiarities of each, let's look at some of the features they all share in common.



The diagram above shows a "ray tracing" image of a concave mirror, showing how a sample ray of light bounces off it. Though we will take this image as an example, the same principles and vocabulary apply to convex mirrors and to lenses as well.

The **principal axis** of a mirror or lens is a normal that typically runs through the center of the mirror or lens. The **vertex**, represented by *V* in the diagram, is the point where the principal axis intersects the mirror or lens.

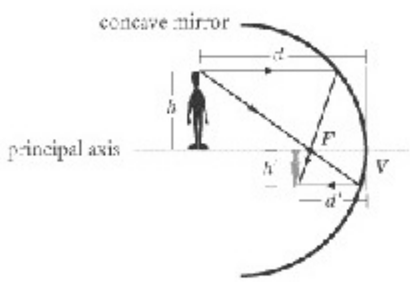
Spherical mirrors have a **center of curvature**, represented by *C* in the diagram, which is the center of the sphere of which they are a slice. The radius of that sphere is called the **radius of curvature**, *R*.

All rays of light that run parallel to the principal axis will be reflected—or refracted in the case of lenses—through the same point, called the **focal point**, and denoted by *F* on the diagram. Conversely, a ray of light that passes through the focal point will be reflected parallel to the principal axis. The **focal length**, *f*,is defined as the distance between the vertex and the focal point. For spherical mirrors, the focal length is half the radius of curvature, $f = R/2$.

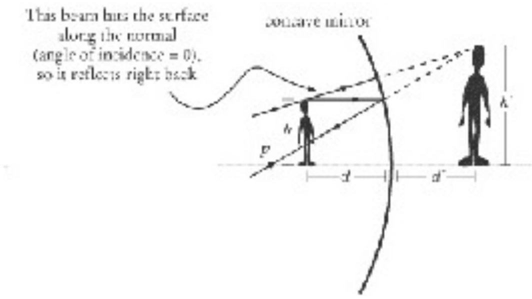
Concave Mirrors

Suppose a boy of height *h* stands at a distance *d* in front of a concave mirror. By tracing the light rays that come from the top of his head, we can see that his reflection would be at a distance d' from the mirror and it would have a height h'. As anyone who has looked into a spoon will have guessed, the image appears upside down.

The image at d' is a **real image**, as we can see from the ray diagram; the image is formed by actual rays of light. It means that, if you were to hold up a screen at position d’, the image of the boy would be projected onto it. You may have noticed the way that the concave side of a spoon can cast light as you turn it at certain angles. That’s because concave mirrors project real images.



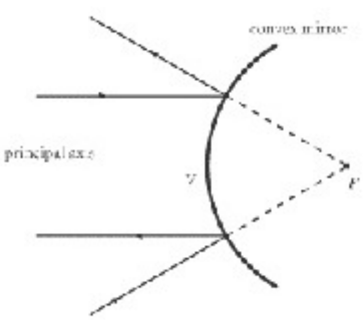
You’ll notice, though, that we were able to create a real image only by placing the boy behind the focal point of the mirror. What happens if he stands in front of the focal point?



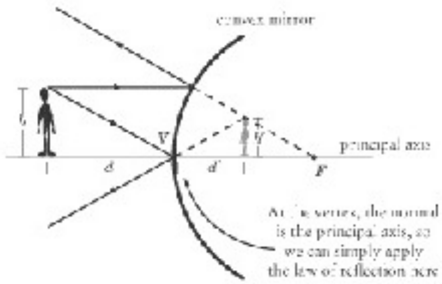
The lines of the ray diagram do not converge at any point in front of the mirror, which means that no real image is formed: a concave mirror can only project real images of objects that are behind its focal point. However, we can trace the diverging lines back behind the mirror to determine the position and size of a **virtual image**. Like an ordinary flat mirror, the image appears to be standing behind the mirror, but no light is focused on that point behind the mirror. With mirrors generally, an image is real if it is in front of the mirror and virtual if it is behind the mirror. The virtual image is right side up, at a distance d' from the vertex, and stands at a height h'.

You can test all this yourself with the right kind of spoon. As you hold it at a distance from your face, you see your reflection upside down. As you slowly bring it closer, the upside-down reflection becomes blurred and a much larger reflection of yourself emerges, this time right side up. The image changes from upside down to right side up as your face crosses the spoon's focal point.

Convex Mirrors



The focal point of a convex mirror is behind the mirror, so light parallel to the principal axis is reflected away from the focal point. Similarly, light moving toward the focal point is reflected parallel to the principal axis. The result is a virtual, upright image, between the mirror and the focal point.



You have experienced the virtual image projected by a convex mirror if you've ever looked into a polished doorknob. Put your face close to the knob and the image is grotesquely enlarged, but as you draw your face away, the size of the image diminishes rapidly.

The Two Equations for Mirrors and Lenses

So far we have talked about whether images are real or virtual, upright or upside down. We've also talked about images in terms of a focal length *f*, distances *d* and d', and heights *h* and h'. There are two formulas that relate these variables to one another, and that, when used properly, can tell whether an image is real or virtual, upright or upside down, without our having to draw any ray diagrams. These two formulas are all the math you'll need to know for problems dealing with mirrors and lenses.

First Equation: Focal Length

The first equation relates focal length, distance of an object, and distance of an image:

$$1/d + 1/d' = 1/f$$

Values of *d*, d', and *f* are positive if they are in front of the mirror and negative if they are behind the mirror. An object can't be reflected unless it's in front of a mirror, so *d* will always be positive. However, as we've seen, *f* is negative with convex mirrors, and d' is negative with convex mirrors and with concave mirrors where the object is closer to the mirror than the focal point. A negative value of d' signifies a virtual image, while a positive value of d' signifies a real image.

Note that a normal, flat mirror is effectively a convex mirror whose focal point is an infinite distance from the mirror, since the light rays never converge. Setting

$1/f = 0$, we get the expected result that the virtual image is the same distance behind the mirror as the real image is in front.

Second Equation: Magnification

The second equation tells us about the **magnification**, *m*, of an image:

$m = h'/h = -d'/d$

Values of h' are positive if the image is upright and negative if the image is upside down. The value of *m* will always be positive because the object itself is always upright.

The magnification tells us how large the image is with respect to the object: if $|m| > 1$, then the image is larger; if $|m| < 1$, the image is smaller; and if $m = 1$, as is the case in an ordinary flat mirror, the image is the same size as the object.

Because rays move in straight lines, the closer an image is to the mirror, the larger that image will appear. Note that d'/d will have a positive value with virtual images and a negative value with real images. Accordingly, the image appears upright with virtual images where *m* is positive, and the image appears upside down with real images where *m* is negative.

Example

A woman stands 40cm from a concave mirror with a focal length of 30cm. How far from the mirror should she set up a screen in order for her image to be projected onto it? If the woman is 1.5cm tall, how tall will her image be on the screen?

How far from the mirror should she set up a screen in order for her image to be projected onto it?

The question tells us that $d = 40$ cm and $f = 30$ cm. We can simply plug these numbers into the first of the two equations and solve for d', the distance of the image from the mirror:

$1/40\text{cm} + 1/d' = 1/30\text{cm}$

$1/d' = 1/120\text{cm}$

d' = 120cm

Because d' is a positive number, we know that the image will be real. Of course, we could also have inferred this from the fact that the woman sets up a screen onto which to project the image.

How tall will her image be on the screen?

We know that *d* = 40 cm, and we now know that d' = 120 cm, so we can plug these two values into the magnification equation and solve for *m*:

m = -d'/d

= -120cm/40cm

= -3

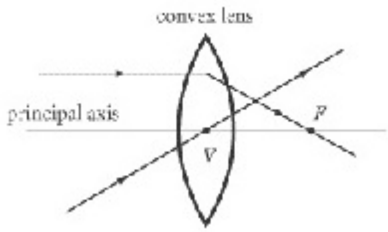
The image will be three times the height of the woman, or 1.5 x 3 = 4.5m tall. Because the value of *m* is negative, we know that the image will be real, and projected upside down.

Convex Lenses

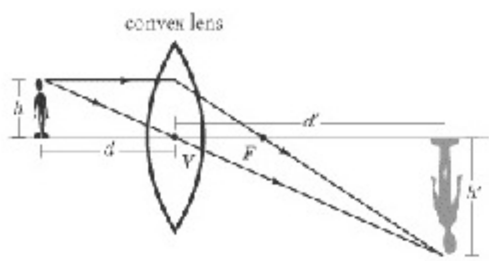
Lenses behave much like mirrors, except they use the principle of refraction, not reflection, to manipulate light. You can still apply the two equations above, but this difference between mirrors and lenses means that the values of d' and *f* for lenses are positive for distances behind the lens and negative for distances in front of the lens. As you might expect, *d* is still always positive.

Because lenses, both concave and convex, rely on refraction to focus light, the principle of dispersion tells us that there is a natural limit to how accurately the lens can focus light. For example, if you design the curvature of a convex lens so that red light is focused perfectly into the focal point, then violet light won't be as accurately focused, since it refracts differently.

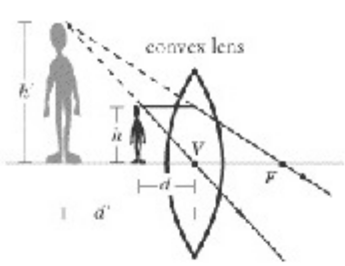
A **convex lens** is typically made of transparent material with a bulge in the center. Convex lenses are designed to focus light into the focal point. Because they focus light into a single point, they are sometimes called "converging" lenses. All the terminology regarding lenses is the same as the terminology we discussed with regard to mirrors—the lens has a vertex, a principal axis, a focal point, and so on.



Convex lenses differ from concave mirrors in that their focal point lies on the opposite side of the lens from the object. However, for a lens, this means that *f* > 0, so the two equations discussed earlier apply to both mirrors and lenses. Note also that a ray of light that passes through the vertex of a lens passes straight through without being refracted at an angle.

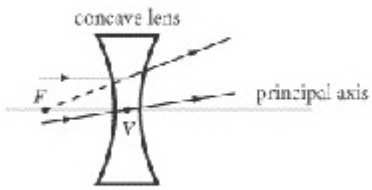


In this diagram, the boy is standing far enough from the lens that *d* > *f*. As we can see, the image is real and on the opposite side of the lens, meaning that d' is positive. Consequently, the image appears upside down, so h' and *m* are negative. If the boy were now to step forward so that *d* < *f*, the image would change dramatically:

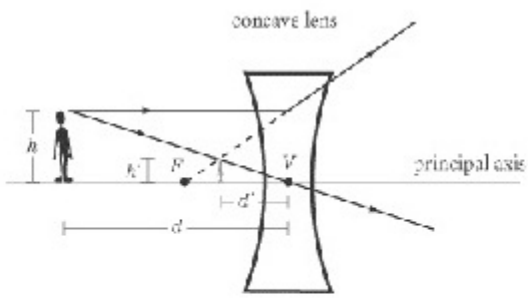


Now the image is virtual and behind the boy on the same side of the lens, meaning that d' is negative. Consequently, the image appears upright, so h' and *m* are positive.

Concave Lenses



A **concave lens** is designed to divert light away from the focal point, as in the diagram. For this reason, it is often called a “diverging” lens. As with the convex lens, light passing through the vertex does not bend. Note that since the focal point *F* is on the same side of the lens as the object, we say the focal length *f* is negative.



As the diagram shows us, and as the two equations for lenses and mirrors will confirm, the image is virtual, appears on the same side of the lens as the boy does, and stands upright. This means that d’ is negative and that h’ and *m* are positive. Note that *h* >h’, so *m* < 1.