Introduction to Straight-Line Graphs

Linear Graphs

A **graph** is a picture that represents numerical data. Most of the graphs that you have been taught are **straight-line** or **linear graphs**. This topic shows how to use linear graphs to represent various real-life situations.

If the rule for a relation between two variables is given, then the graph of the relation can be drawn by constructing a table of values.

To plot a **straight line graph** we need to find the coordinates of *at least two points* that fit the rule.

Example

Plot the graph of $y = 3x + 2$.

Solution

Construct a table and choose simple x values.

X	-2	-1	0	1	2
Y					

In order to find the y values for the table, substitute each x value into the rule y = 3x + 2

When x = -2, y = 3(-2) + 2

= -6 + 2 = = -4

When x = -1, y = 3(-1) + 2

= -3 + 2 = 1

When x = 0, y = 3 X 0 + 2

= 0 + 2 = 2

When x = 1, y = 3 X 1 + 2

= 3 + 2 = 5

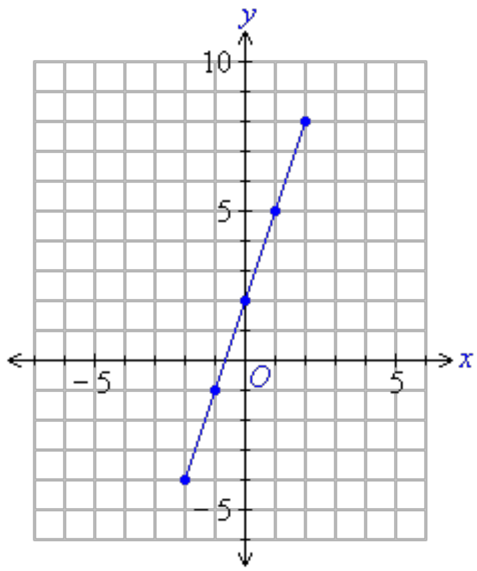
When x = 2, y = 3 X 2 + 2

= 6 + 2 = 8

The table of values obtained after entering the values of y is as follows:

X	-2	-1	0	1	2
Y	-4	-1	2	5	8

Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



Setting out:

Often, we set out the solution as follows.

Y = 3x + 2

When x = -2, y = 3(-2) + 2

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When x = -1, y = 3(-1) + 2

= -3 + 2 = = -1

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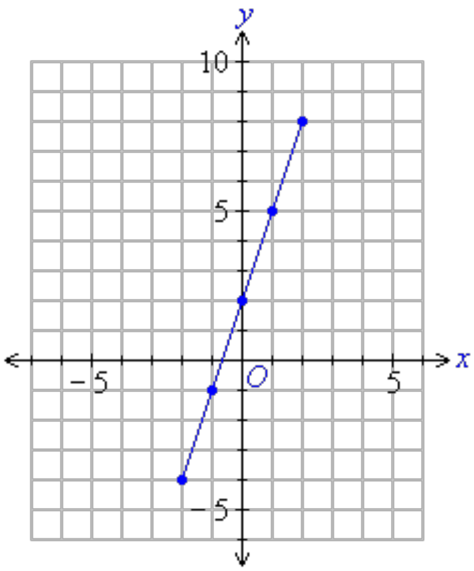


Example

Plot the graph of $y = -2x + 4$.

Solution

Y = -2x + 4

When x = -2, y = -2(-2) + 4

= 4 + 4 = 8

When x = -1, y = -2(-1) + 4

= 2 + 4 = 6

When x = 0, y = -2 X 0 + 4

= 0 + 4 = 4

When x = 1, y = -2(1) + 4

= -2 + 4 = 2

When x = 2, y = -2(2) + 4

= -4 + 4 = 0

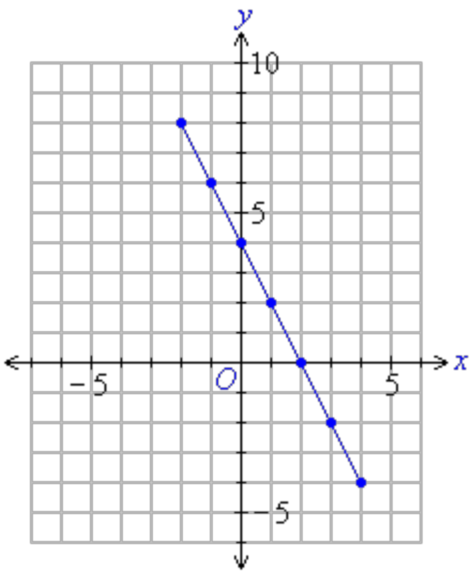
When x = 3, y = -2(3) + 4

= -6 + 4 = -2

When x = 4, y = -2(4) + 4

= -8 + 4 = -4

x	-2	-1	0	1	2	3	4
y	8	6	4	2	0	-2	-4

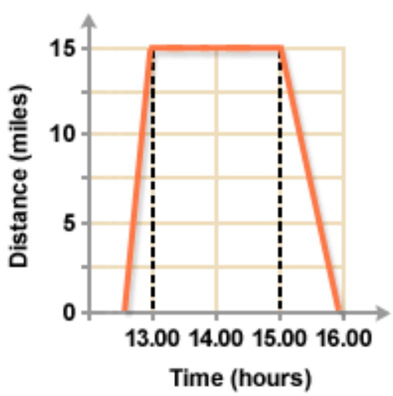


Distance-time graphs

We use distance-and-time graphs to show journeys. It is always very important that you read **all** the information shown on these type of graphs.

A graph showing one vehicle's journey

If we look at the graph shown below, you can see that the time in hours is along the horizontal, and the distance in miles is on the vertical axis. This graph represents a journey that Jan took, in travelling to Glasgow and back, from Aberdeen.



Important points to note are:

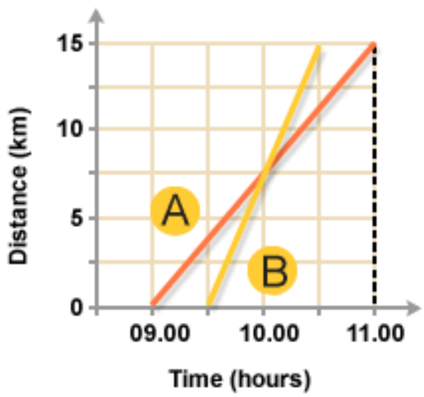
It took half an hour to travel a distance of miles

Between 1pm and 3pm there was no distance travelled. This means that the car had stopped.

The journey back, after 3pm, took one hour.

A graph showing two different journeys in the same direction

The next graph shows two different journeys. You can see that there is a difference with the steepness of the lines drawn. Remember that, the steeper the line, the faster the average speed. We can calculate the average speeds, by reading distances from the graph, and dividing by the time taken.



Line A: How long does journey A last, and what distance is travelled?

The journey takes 2 hours, and the distance travelled is 15km.

Line B : How long does journey B last, and what distance is travelled?

The journey takes 1 hour, and the distance travelled is also 15 km.

This means that the average speeds are:

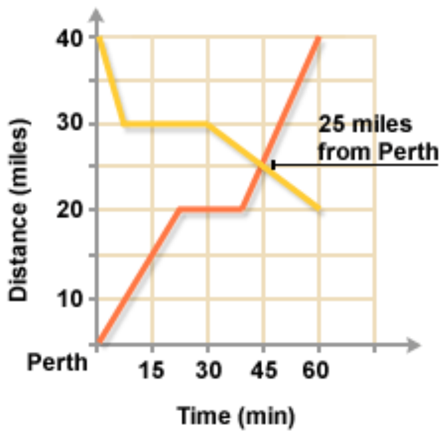
A: 15/2 = 7.5 km per hour

B: 15/1 km per hour

You will also notice from the graph that the two lines cross. This means that, if the two vehicles were travelling along the same route, they would have met at that point, which was just after 10am. The vehicle on journey B overtook vehicle

A graph showing two journeys in the opposite direction

A different pair of journeys is shown below. It is important to note that one journey begins at a distance of , and the other at a distance of miles, from Perth. In fact, what is happening is that one journey is travelling away from, and the other is travelling towards Perth. Again, the two journeys meet. This time it is miles from Perth.



You will also see that the two journeys contain stops.
If we were to calculate the average speeds for each total journey we would have to include this time as well.
A graph showing a journey (or journeys) should have time on the horizontal axis, and distance from somewhere on the vertical axis.
A line moving up, as it goes from left to right, shows a journey moving away from a place, and a line moving down, as it goes from left to right, represents a journey towards a place.
A horizontal line is a break or rest.
Two lines, sloping the same way, cut: then an overtaking has taken place.
Two lines, sloping opposite ways, cut: a meeting has taken place.

Speed-time graphs

A **speed-time graph,** velocity-time graph, shows how the speed of an object varies with time during a journey.
There are two very important things to remember about velocity – time graphs.
The distance travelled is the area under the graph.
The gradient or slope of the graph is equal to the acceleration. If the gradient is negative, then there is a deceleration. We may use the equations(1) or some rearrangement of this equation.
Example. A car starts on a journey. It accelerates for 10 seconds at It then travels at a constant speed for 50 seconds before coming to rest in a further 4 seconds.
a. Sketch a velocity – time graph.
b. Find the total distance travelled.
c. Find the deceleration when the car is coming to a stop at the end.

d. Find the average speed.

a. We may rearrange (1) to obtain v = u + at = 0 + 3 X 10 = 30m/s. Hence we may draw a straight line from (0,0) to (3,30). During the second part the car is travelling at a constant speed of 30m/3. Hence we can draw a straight line to (3,30)+(50,0)=(53,30). During the last part, which takes a further 4 seconds the car comes to a rest, and it's final velocity will be zero. Hence we can draw a straight line to (57,0). We can now draw the velocity time graph.

b. Distance travelled = Area under the graph. The graph is a trapezium so use the formula for the area of a trapezium: ½(a + b) X h = 1/2 (57 + 50) X 30 = 1535m

c. During the final part of the journey the velocity decreases from 30 to 0 in 4 seconds so a = v – u/t = (0 – 30)/4 = -7.5m/s^2 → deceleration = 7.5m/s^2

d. Average speed = Total Distance/Total Time = 1535/57 = 26.93m/s^2.

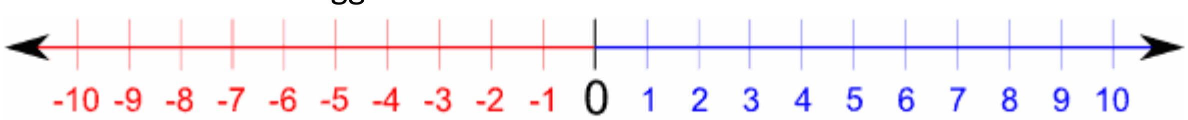
GRAPHS – CARTESIAN PLANE AND COORDINATES

The position of points

A **graph** is a picture of numerical data. We used graphs in statistics in class 1, where they represented number patterns. Here we extend graphs to identifying and drawing the position of points.

Points on a line

Writing numbers down on a Number Line makes it easy to tell which numbers are bigger or smaller.



Numbers on the left are stronger than numbers on the right.

Example

5 is smaller than 8

-1 is smaller than 1

-8 is smaller than -5

Example

Example: John owes \$3, Virginia owes \$5 but Alex doesn't owe anything, in fact he has \$3 in his pocket. Place these people on the number line to find who is poorest and who is richest.

Having money in your pocket is positive, owing money is negative.

So John has “−3”, Virginia “−5” and Alex “+3”

Now it is easy to see that Virginia is poorer than John (−5 is less than −3) and John is poorer than Alex (−3 is smaller than 3), and Alex is, of course, the richest!

Plotting Points on a Cartesian Plane

A Cartesian plane (named after French mathematician Rene Descartes, who formalized its use in mathematics) is defined by two perpendicular number lines: the ***x*-axis**, which is horizontal, and the ***y*-axis**, which is vertical. Using these axes, we can describe any point in the plane using an ordered pair of numbers.

The Cartesian plane extends infinitely in all directions. To show this, math textbooks usually put arrows at the ends of the axes in their drawings.

The location of a point in the plane is given by its coordinates, a pair of numbers enclosed in parentheses: (x, y). The first number x gives the point’s horizontal position and the second number y gives its vertical position. All positions are measured relative to a “central” point called the origin, whose coordinates are (0, 0). For example, the point (5, 2) is 5 units to the right of the origin and 2 units up, as shown in the figure. Negative coordinate numbers tell us to go left or down. See the other points in the figure for examples.

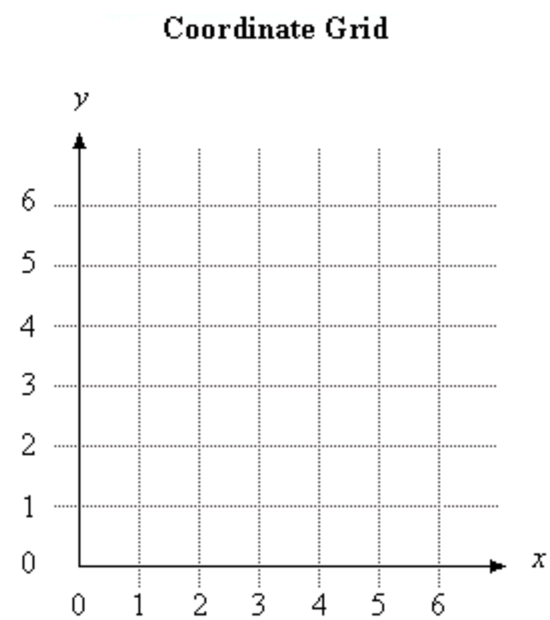
The Cartesian plane is divided into four quadrants. These are numbered from I – IV, starting with the upper right and going around counterclockwise. (For some reason everybody uses roman numerals for this).

In Quadrant I, both the x– and y-coordinates are positive; in Quadrant II, the x-coordinate is negative, but the y-coordinate is positive; in Quadrant III both are negative; and in Quadrant IV x is positive but y is negative.

Points which lie on an axis (i.e., which have at least one coordinate equal to 0) are said not to be in any quadrant. Coordinates of the form $(x, 0)$ lie on the horizontal x-axis, and coordinates of the form $(0, y)$ lie on the vertical y-axis.

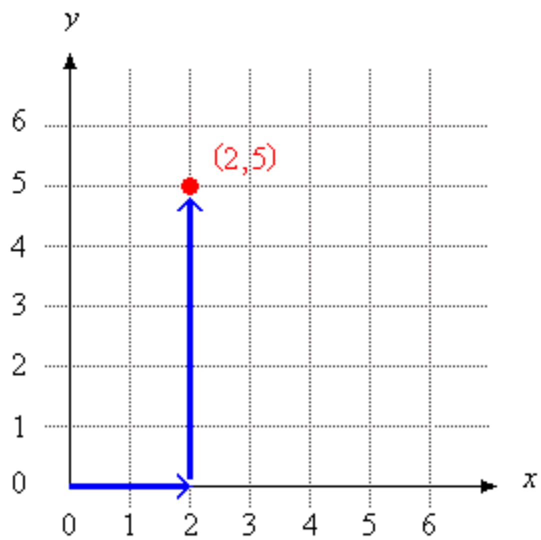
Coordinate Graphing

Coordinate graphing sounds very dramatic but it is actually just a visual method for showing relationships between numbers. The relationships are shown on a **coordinate grid**. A coordinate grid has two perpendicular lines, or **axes**, labeled like number lines. The **horizontal axis** is called the ***x*-axis**. The **vertical axis** is called the ***y*-axis**. The point where the *x*-axis and *y*-axis intersect is called the **origin**.

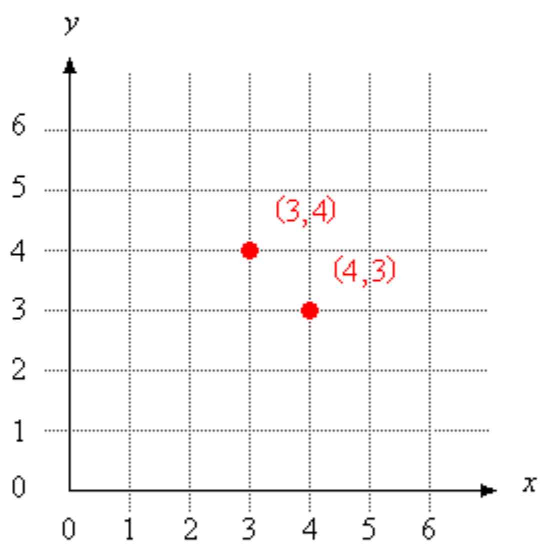


The numbers on a coordinate grid are used to locate points. Each point can be identified by an **ordered pair** of numbers; that is, a number on the *x*-axis called an ***x*-coordinate**, and a number on the *y*-axis called a **y-coordinate**. Ordered pairs are written in parentheses (*x*-coordinate, *y*-coordinate). The origin is located at (0,0). Note that there is no space after the comma.

The location of (2,5) is shown on the coordinate grid below. The *x*-coordinate is 2. The *y*-coordinate is 5. To locate (2,5), move 2 units to the right on the *x*-axis and 5 units up on the *y*-axis.



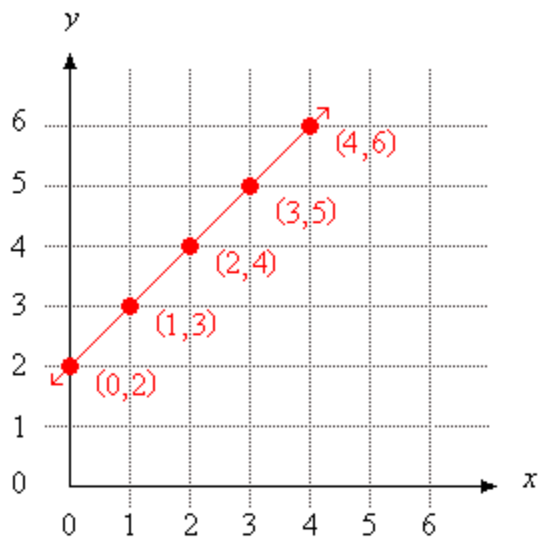
The order in which you write *x*– and *y*-coordinates in an ordered pair is very important. The *x*-coordinate always comes first, followed by the *y*-coordinate. As you can see in the coordinate grid below, the ordered pairs (3,4) and (4,3) refer to two different points!



The function table below shows the *x*– and *y*-coordinates for five ordered pairs. You can describe the relationship between the *x*– and *y*-coordinates for each of these ordered pairs with this rule: the x-coordinate plus two equals the y-coordinate. You can also describe this relationship with the algebraic equation $x + 2 = y$.

***x*-coordinate**	***x* + 2 = *y***	***y*-coordinate**	**ordered pair**
0	**0** + 2 =**2**	2	(0,2)
1	**1** + 2 =**3**	3	(1,3)
2	**2** + 2 =**4**	4	(2,4)
3	**3** + 2 =**5**	5	(3,5)
4	**4** + 2 =**6**	6	(4,6)

To graph the equation $x + 2 = y$, each ordered pair is located on a coordinate grid, then the points are connected. Notice that the graph forms a straight line. The arrows indicate that the line goes on in both directions. The graph for any simple addition, subtraction, multiplication, or division equation forms a straight line.



Plotting Points

To plot a point means to raw its position on a Cartesian plane. The easiest way to plot a point is as follows:

1. Start at the origin.
2. Move along the x-axis by an amount and in a direction given by the x-coordinate of the point.
3. Move up or down parallel to the y-axis by an amount and in a direction given by the y-coordinate.