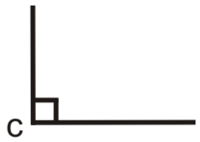
MATHEMATICS

BASIC THEOREMS OF ANGLES/TRIANGLE

Angles are the distances or changes in direction between two lines or surface diverging from the same point. It is measured in degree (^{0}).

Types of angles

a. **Acute angle**: This is an angle that is less than 90^0.
b. **Right angle**: This is an angle that is exactly 90^0 or a greater of a resolution.



c. **Obtuse angle**: This is an angle that is greater than 90^0 but less than 180^0

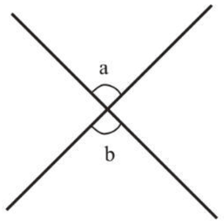
d. **Angle on a straight line**: This is an angle that is exactly 180 or two right angle

e. **Reflex angle**: This is an angle that is greater than 180 but less than 360.

f. **Angle at a point**: This is exactly four right angles (360) $a + b + c + d = 360^0$.

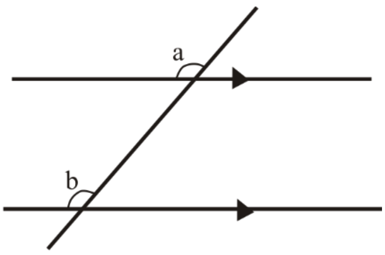
Basic Theorems

1) Vertically opposite angles are equal



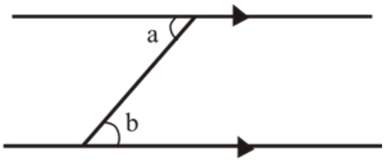
$a = b$

2) Corresponding angles are equal



$a = b$

3) Alternate angles are equal

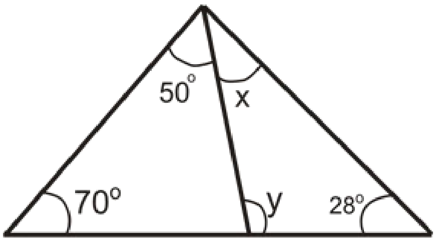


a = b

Note: The sum of angle of a triangle is 180°

Example

Find the values of each of the angles marked below



Solution:

50 + 70 + z =180 (sum of angle on a triangle)

Z = 180-70-50

Z = 60^0

60 + y = 180 (angle on a straight line)

y = 180-60

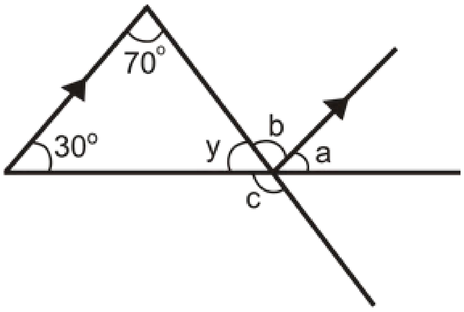
y = 120

120 + 28 + x = 180 (sum of angles on a triangle)

X = 180-120-28

X = 32^0

Example 2



70 + 30 + y = 180 (sum of angles on a triangle)
Y +100 =180
y = 180 -100
y = 80°

b = 70° (alternate angle)

c + y =180 (sum of angle on a straight line)
Where y = 80°
→ c + 80 = 180
c = 180⁰ – 80°
c = 100°

a + b + y = 180° (sum of angles on a straight line)
Where b = 70° and y = 80°
→ a + 70° + 80° = 180°
a + 150° = 180°
a = 180° – 150°
a = 30°

Example 3
What is the sum of the interior angle of a duodecagon polygon?

Solution:
A duodecagon polygene has 12 side: n = 12
Therefore, sum of its interior angle = (2n – 4) x 90°

= (2x12 – 4)

= (24 – 4) x 90°

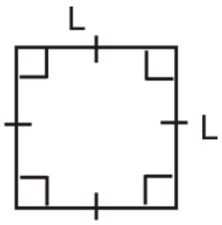
20 x 90°

1800°

Basic geometric thermos parallelogram and equal intercept

A quadrilateral is a polygon with four sides.
Types of quadrilateral include the square, rectangle, parallelogram, rhombus, kite and trapezium.

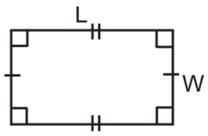
1. **Square**



Properties

i. All four sides are equal.
ii. The sum of the angles in a square is 360.
iii. It has four right angles.
iv. The opposite side are parallel.

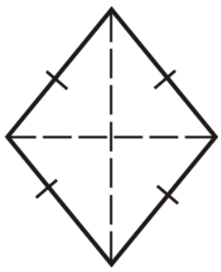
2. **Rectangle**



Properties

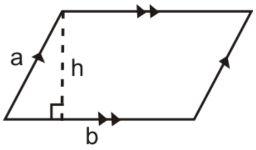
i. The opposite sides are equal and parallel.
ii. It has four right angles.
iii. The diagrams are equal in length.

3. **Rhombus**



i. All the four sides are equal.
ii. The opposite sides are parallel and equal in length.
iii. The opposite angles are equal.

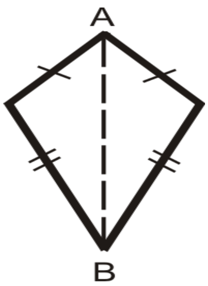
4. **Parallelogram**



Properties:

i. The opposite side are parallel and equal in length.
ii. The opposite angles are equal.
iii. The diagonals bisect each other but they are not equal in length.

Kite:

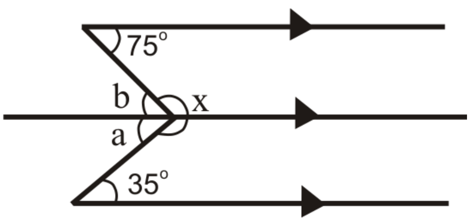


Properties:

i. Two pairs of adjacent sides are equal in length.
ii. One of the diagonals bisects the other, but they are not equal in length.
iii. The diagonal intersect at right angle.

Example 1

Calculate the value of x



a = 35o (alternate angle) and b = 75 (alternate angles) hence a + b + x = 360 (sum of angle of a point)

$35^o + 75^o + x = 360^o$

$x = 360^o - (35 + 75)^o$

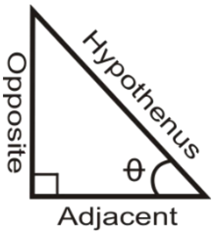
$= 360^o – 110^o$

$= 250^o$

TRIGONOMETRY 1

(Sine, cosine and tangent)

Trigonometric ratios: **SOH CAH TOA** (can be used to remember each of the ratios)



First; **SOH:** Sinθ = $\frac{\text{Opposite}}{\text{Hypothenus}}$

Second**; CAH:** Cosθ = $\frac{\text{Adjacent}}{\text{Hypothenus}}$

Third**; TOA:** Tanθ = $\frac{\text{Opposite}}{\text{Adjacent}}$

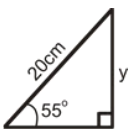
Example:
Find the sine, cosine and tangent of the following angle using *four figure table.*
(a.) 41^0 (b) 62^0

Solution
a) Sin 41^o = 0.6561
Cos 41^o = 0.7547
Tan 41^o = 0.8693

b) Sin 62^o = 0.8829
Cos 62^o = 0.4695
Tan 62^o = 1.881

Note: you must be careful when using the logarithm table to avoid looking for answer in the wrong place

2. Calculate the value of y in the diagram below.



Solution
Using **SOH CAH TOA**
$\frac{\text{Sin } 55}{1} = \frac{y}{20}$
Cross-multiplying:

$y = 20 \times Sin55^o$
$y = 16.384cm$

Example 3

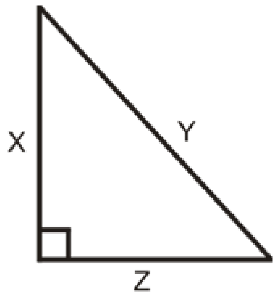
Calculate θ , if sin θ = cos θ

Solution:
Sin θ = cos (90 – θ)
Therefore cos (90 – θ) = cos θ
Therefore 90 – θ = θ
90 = 2θ
θ = 45

Trigonometry (solving problem on right angle triangles)

Pythgoram thermos
There are relationship between the sides and angle of angle trangle. In a right angle triangle, the relationship is the trigonometric ratios of sine, cosine and tangent
Sin θ = opp/hyp. Cos θ= adj/hyp, tan θ = opp.adj

Pythegoras theorem
The square on the hypotense of a right angle triangle is equal to the sum of the squares of the other two sides.

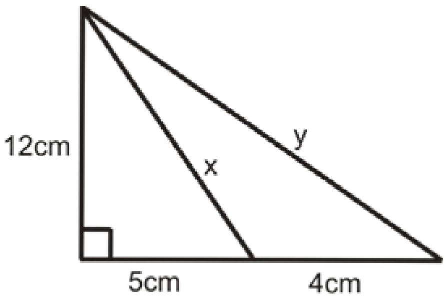


$Y^2 = x^2+z^2$

$X^2 = y^2-z^2$

$Z^2 = y^2- x^2$

Example 1



Calculate the sides marked x and y on the diagram

x ^{2} = $\sqrt{12^2 + 5^2}$

$x^2 = \sqrt{144 + 25}$

$x^2 = \sqrt{169}$

x = 13cm

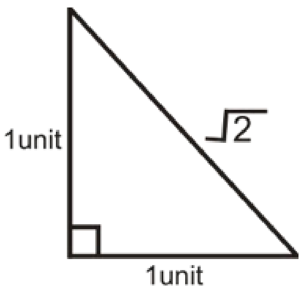
$y^2 = \sqrt{12^2 + 9^2}$
$y^2 = \sqrt{144 + 81}$
$y^2 = \sqrt{225}$
Therefore, y = 15cm

SPECIAL ANGLES

45^o, 30^o and 60^o

Deriving special angles is an approach which helps ud to abtain trigonometric ratios of angle without using the trigonometric tables or other calculating aids

Ratios for 45^o



Sin 45 = $\frac{1}{\sqrt{2}}$

Cos 45 = $\frac{1}{\sqrt{2}}$

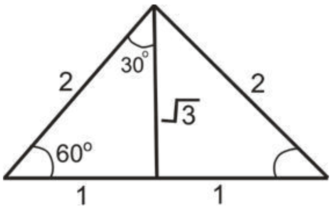
Tan 45 = $\frac{1}{1}$ =1

Cosec45 = $\frac{\sqrt{2}}{1}$ = $\sqrt{2}$

Cot45 = $\frac{1}{1}$ =1

Sec45 = $\frac{\sqrt{2}}{1}$ = $\sqrt{2}$

Ratio from 30 and 60



Angle 30°

Sin 30 = $\frac{1}{2}$

Cos 30 = $\frac{\sqrt{3}}{2}$

Tan 30 = $\frac{1}{\sqrt{3}}$

Angle 60°

Sin 60^0 = $\frac{\sqrt{3}}{2}$

Cos 60° = $\frac{1}{2}$

Tan 60° = $\frac{\sqrt{3}}{1}$ = $\sqrt{3}$

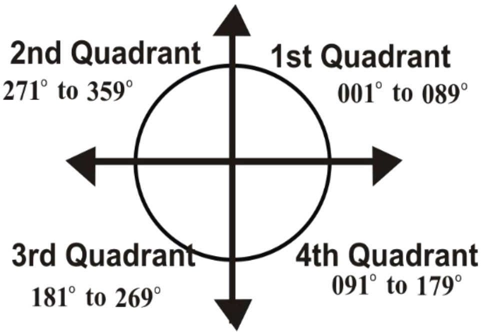
Angle 90°

Consider a hypothetical triangle with the angles = 90 and sides I unit
Sin 90 = 1/1 = 1
Cos 90 = 0/1
Tan 90 = 1/0 = ∞

TRIGONOMETRY (2)

(General angles and graphs)

General angles are angles of sizes between 0 and 360.



Example 1

Locate the quadrants of the following angles

a. 35
b. 195
c. -18

Solution

a. 35 is in the 1^{st} quadrant.
b. 195 is located in 3^{rd} quadrant.
c. -18 is located in 4^{th} quadrant.

Example 1

Find the equivalents of the following angle

a. 30
b. 120

Solution

a. 30 is in the 1^{st} quadrate
  Therefore sin 30 = sin 30
  Cos 30 = cos 30
  Tan 30 = tan 30

b. 120 = (180- 60)
Therefore sin 120 = sin 60
Cos 120 = cos 60
Tan 120 = - tan 60

Example 3

If sin θ = 3/5 find cos θ and tanθ, if 0 < θ < 90^o

Solution

Sinθ = opp/hyp = 3/5

Using Pythagoras theorem

x ^{2} = $\sqrt{5^2 - 3^2}$

x = $\sqrt{25 - 9}$

x = $\sqrt{16}$

Therefore, x =4

This implies that; cos θ = 4/5

And tan θ = 3/4